# 電磁気学II演習 解答 No.7

1. No.4 の1と同様に $E$ を求める。

(i) $r \le R$ (球内部)

$$\int_S \left\{ \vec{E}(\vec{r}) \cdot \vec{n}(\vec{r}) \right\} \mathrm{d}S = \frac{1}{\varepsilon_0} \cdot \frac{4}{3} \pi r^3 \rho$$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{\rho \left( x^2 + y^2 + z^2 \right)^{1/2}}{3\varepsilon_0}$$

$$\vec{E}\left(\vec{r}\right) = \frac{\rho}{3\varepsilon_0} \vec{r} = \frac{\rho}{3\varepsilon_0} \left( x, y, z \right)$$

$$\frac{\partial E_x}{\partial x} = \frac{\partial E_y}{\partial y} = \frac{\partial E_z}{\partial z} = \frac{\rho}{3\varepsilon_0}$$

$$\therefore \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \times \left( \frac{\rho}{3\varepsilon_0} x, \frac{\rho}{3\varepsilon_0} y, \frac{\rho}{3\varepsilon_0} z \right) = 0$$

(ii) $r > R$ (球外部)

$$E\left(r\right) = \frac{1}{4\pi\varepsilon_0 r^2} \frac{4}{3} \pi R^3 \rho = \frac{\rho R^3}{3\pi\varepsilon_0 r^2}$$

$$\vec{E}\left(\vec{r}\right) = \frac{\rho R^3}{3\pi\varepsilon_0 r^3} \vec{r}$$

$$\frac{\partial E_x}{\partial x} = \frac{\partial}{\partial x} \left\{ \frac{\rho R^3}{3\pi\varepsilon_0 r^3} \cdot x \right\} = \frac{\rho R^3}{3\varepsilon_0} \left\{ \left( -\frac{3x^2}{r^5} + \frac{1}{r^3} \right) \right\} = \frac{\rho R^3}{3\varepsilon_0} \left( \frac{1}{r^3} - \frac{3x^2}{r^5} \right),$$

$$\frac{\partial E_y}{\partial y} = \frac{\rho R^3}{3\varepsilon_0} \left( \frac{1}{r^3} - \frac{3y^2}{r^5} \right), \qquad \frac{\partial E_z}{\partial z} = \frac{\rho R^3}{3\varepsilon_0} \left( \frac{1}{r^3} - \frac{3z^2}{r^5} \right).$$

$$\left( \vec{\nabla} \times \vec{E} \right)_x = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} = \frac{\partial}{\partial z} \left( \frac{\rho R^3}{3\varepsilon_0} \frac{y}{r^3} \right) - \frac{\partial}{\partial y} \left( \frac{\rho R^3}{3\varepsilon_0} \frac{z}{r^3} \right) = \frac{\rho R^3}{3\varepsilon_0} \left( -\frac{3yz}{r^5} + \frac{3yz}{r^5} \right) = 0$$

$\vec{\nabla} \times \vec{E}$ の $y$、$z$、成分も同様

$$\therefore \vec{\nabla} \times \vec{E} = 0$$

2. 下図のように $x$ 軸、原点 O をとる。電場 $E$ は $x$ 軸方向を向くから、微分形Gaussの法則より

$$|x| > d \qquad \frac{\mathrm{d}E(x)}{\mathrm{d}x} = 0 \qquad \cdots\cdots\cdots ①$$

$$|x| \le d \qquad \frac{\mathrm{d}E(x)}{\mathrm{d}x} = \frac{\rho}{\varepsilon_0} \qquad \cdots\cdots\cdots ②$$

①より $x > d$、$x < -d$ の領域で電場は一定だから、$x > d$ で $E(x) = E$ とおけば、$x < -d$ で $E(x) = -E$

②を積分すると $-d \le x \le d$ の領域では

$$E(x) = \frac{\rho x}{\varepsilon_0} + C$$

$E(x)$ は $x = \pm d$ で連続だから

$$\frac{\rho d}{\varepsilon_0} + C = E \qquad (x = d)$$

$$-\frac{\rho d}{\varepsilon_0} + C = -E \qquad (x = -d)$$

より、 $E = \dfrac{\rho d}{\varepsilon_0},\ C = 0$

$$\therefore E(x) = \begin{cases} -\rho d/\varepsilon_0 & (x < -d) \\ \rho x/\varepsilon_0 & (-d \le x \le d) \\ \rho d/\varepsilon_0 & (x > d) \end{cases}$$