電磁気学II演習 解答 No.5

1. $\phi(\vec{r}) = \dfrac{q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|}$ より

$$\phi(x,y,z) = \frac{q}{4\pi\varepsilon_0}\underline{\left\{\frac{1}{\left[x^2+y^2+(z-d)^2\right]^{1/2}} - \frac{1}{\left[x^2+y^2+(z+d)^2\right]^{1/2}}\right\}} \text{[i]}$$

$$\left[x^2+y^2+(z\mp d)^2\right]^{-1/2} \cong \left(x^2+y^2+z^2\mp 2dz\right)^{-1/2}$$

$$= \left(r^2 \mp 2dz\right)^{-1/2} = \frac{1}{r}\left(1\mp\frac{2dz}{r^2}\right)^{-1/2} \cong \underline{r^{-1}\left(1\pm\frac{zd}{r^2}\right)} \text{[ii]}$$

$\phi(x,y,z)$に代入すると

$$\phi(x,y,z) \cong \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1+\frac{dz}{r^2}\right) - \frac{1}{r}\left(1-\frac{dz}{r^2}\right)\right]$$

$$= \frac{2qzd}{4\pi\varepsilon_0 r^3} = \underline{\frac{p}{4\pi\varepsilon_0}}\frac{z}{r^3} \text{[iii]}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r^3}\right) = \frac{\partial}{\partial x}\left\{\left(x^2+y^2+z^2\right)^{-3/2}\right\} = -\frac{3}{2}\left(x^2+y^2+z^2\right)^{-5/2}\cdot 2x$$

$$= \underline{-\frac{3x}{r^5}} \text{[iv]}$$

$$\frac{\partial}{\partial y}\left(\frac{1}{r^3}\right) = \underline{-\frac{3y}{r^5}} \text{[v]}, \quad \frac{\partial}{\partial z}\left(\frac{1}{r^3}\right) = \underline{-\frac{3z}{r^5}} \text{[vi]}$$

よって

$$E_x = -\frac{\partial\phi}{\partial x} = -\frac{2qd}{4\pi\varepsilon_0}\frac{\partial}{\partial x}\left(\frac{z}{r^3}\right) = -\frac{2qd}{4\pi\varepsilon_0}\frac{3xz}{r^5} = \underline{\frac{p}{4\pi\varepsilon_0}}\frac{3xz}{r^5} \text{[vii]}$$

$$E_y = -\frac{\partial\phi}{\partial y} = -\frac{2qd}{4\pi\varepsilon_0}\frac{3yz}{r^5} = \underline{\frac{p}{4\pi\varepsilon_0}}\frac{3yz}{r^5} \text{[viii]}$$

$$E_z = -\frac{\partial\phi}{\partial z} = -\frac{2qd}{4\pi\varepsilon_0}\frac{\partial}{\partial z}\left(\frac{z}{r^3}\right) = -\frac{2qd}{4\pi\varepsilon_0}\left\{\frac{1}{r^3} + z\frac{\partial}{\partial z}\left(\frac{1}{r^3}\right)\right\}$$

$$= \frac{2qd}{4\pi\varepsilon_0}\frac{3z^2-r^2}{r^5} = \underline{\frac{p}{4\pi\varepsilon_0}\frac{3z^2-r^2}{r^5}} \text{[ix]}$$

2. 直線上の点電荷から離れた点における静電ポテンシャルは

$$\phi(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{r}-\frac{q}{r-d}-\frac{q}{r+d}\right)$$

第2、第3項に

$$(r\pm d)^{-1}=\frac{1}{r}\left(1\pm\frac{d}{r}\right)^{-1}\cong\frac{1}{r}\left[1\mp\frac{d}{r}+\left(\frac{d}{r}\right)^2\right]$$

を用いると

$$\phi(r)=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left\{2-1+\frac{1}{r}-\left(\frac{d}{r}\right)^2-1-\frac{1}{r}-\left(\frac{d}{r}\right)^2\right\}$$

$$=\underline{\frac{-qd^2}{2\pi\varepsilon_0 r^3}}$$

3. 下図のように半径r、長さlの円筒面を考える。

S_1：側面、S_2：上底面、S_3：下底面。

電気力線は電荷の分布する直線に垂直に生じるから

$$\vec{n}_1 // \vec{E}_{S1},\quad \vec{n}_2 \perp \vec{E}_{S2},\quad \vec{n}_3 \perp \vec{E}_{S3}$$

つまり側面だけを考えればよい。

$$E\cdot S_1=\frac{\lambda l}{\varepsilon_0}\quad\Leftrightarrow\quad E\cdot 2\pi rl=\frac{\lambda l}{\varepsilon_0}$$

$$\therefore E=\underline{\frac{\lambda}{2\pi\varepsilon_0 r}}$$